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Thermal Conduction and Economic Equilibrium

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Abstract

Following a multidisciplinary approach and starting from the fact that thermodynamics, a fundamental branch of physics, which considers the science of large systems in equilibrium, we propose to adopt its universal characteristics to the field of economics and more particularly the law of supply and demand. The striking analogy between thermal equilibrium and supply-demand equilibrium stems from the fact that economics as a branch that studies the metabolism of human society as much as a dissipative structure [1, 2, 3] must follow the laws of thermodynamics. This proposition consists, on the one hand, in simulating between the concepts of temperature, heat quantity and thermal conductivity with the economic concepts of price, product quantity and the positive supply shock. On the other hand, to introduce the new concept of "the economic conduction" and to demonstrate that Fourier's law is applicable as well as in economics as in physics.

Key words: *Econophysics, Economic equilibrium,Supply, Demand, Price,Thermo-economics, Thermal Conduction, Temperature, Quantity of heat .*

PACS *numbers :89.65.Gh; 89.75.-k; 89.20.-a ;44.10.+i*

Résumé

En suivant une approche multidisciplinaire utilisant la thermodynamique, branche fondamentale de la physique qui s'intéresse à la science des grands systèmes en équilibre, nous proposons d'adapter ses caractéristiques universelles au domaine de l'économie et plus particulièrement à la loi de l'offre et de la demande. L'analogie frappante entre l'équilibre thermique et l'équilibre offre-demande vient du fait que l'économie, comme branche qui étudie le métabolisme de la société humaine autant qu'une structure dissipative, doit suivre les lois de la thermodynamique[1, 2, 3]. Cette proposition consiste, d'une part, à effectuer une simulation entre les concepts de température, de quantité de chaleur et de conductivité thermique avec les concepts économiques de prix, de quantité de produit et de choc positif d'offre. D'autre part, à introduire le nouveau concept de"conduction économique" et à démontrer que la loi de Fourier est applicable aussi bien en économie qu'en physique.

Mots clés: *Econophysique, Equilibre Economique,Offre, Demande, Prix,Thermo-economie, Conduction Thermique, Temperature, Quantité de chaleur .*

I. Introduction

The application of physics concepts to understand economic behavior has been the main interest of physicists and economists for more than a hundred years. Early economists like Adam Smith have modeled their economic theories on the basis of physical thoughts of their times [4]. Force and rarity are considered by Walras in 1909 as vectors and energy and utility as scalar [5] .Conservation of energy has been used to formulate conservation of economic value [6]. Alfred Marshall, in the beginning of the past century, applied the physical concepts of thermodynamic equilibrium to develop a new theory according to which an economic system reaches equilibrium in an analog form to Maxwell and Boltzmann gases [7]. Paul Samuelson introduced the Le Chatelier principle of thermodynamics to economic theory [8]. Georgescu-Roegen has introduced entropy to economics [9]. In his book More Heat than Light, Philip Mirowski has even suggested Economics as Social Physics and Physics as Nature's Economics [10]. In 1900, Louis Bachelier wrote his doctorate thesis "Theory of Speculation" [11], in which he employed physical concepts such as the diffusion theory and applied ideas based on the Brownian motion to describe the formation of stock market prices (MANTEGNA and STANLEY, 2000) [12]. Newtonian physics and more specifically Bernoulli's theorem, has been used to simulate the equilibrium that is established between consumption (as Demand) and production (as Supply) [13].

In the same way, the concept of "thermo-economics" was introduced by the physicist François Roddier in 2015 in his paper "the thermodynamics of economic transitions" [3]. It describes economic exchanges by assigning to each production two potentials similar to Gibbs Duhem's potentials which is pressure and temperature. Taking account the assignment of the thermal temperature to the notion of price, we suggest in this contribution, a reflection on one of the important aspects of the use of thermodynamics in understanding the mechanisms of the laws of supply and demand. We will approach it from the particularities of the conduction thermal transfer mode, in particular the Fourier law [14], very famous in thermodynamics for the various applications associated with it.

Our approach to establish the analogy between physical concepts and those associated with economics is based on a need to see both physical and economic aspects from the same angle, which spontaneously places us in a universality approach according to which we can simulate, for example, a gravitational law between two masses interacting at an electrostatic force between two electrical charges in a given space. The objective of this work is to discuss another form of thermodynamic adaptation to the economy, namely the thermal temperature to which it is proposed to assign the notion of price.

This paper is organized as follows. In the next section we make a brief definition of the mode of heat transfer by conduction and will enunciate the Fourier law. In Section 2 we will open a parenthesis about the mechanism of supply-demand and explain how the market reaches the equilibrium situation. In Section 3 we will adapt the characteristics of the conduction mode to the market equilibrium. We finish with some conclusions in the last section.

II. Physical Parenthesis: Thermal Conduction

What is thermal conduction? In his work "Analytical Theory of Heat" Joseph Fourier described the phenomenon of thermal conduction through the Fourier law [14]. According to this law, when two systems are at different temperatures, the hottest system yields heat to the coldest. The heat flow created is proportional to the temperature gradient and flows in the direction of decreasing temperatures. In this case Fourier's law is written in the following form:

$$\vec{\phi} = -\lambda \overrightarrow{\text{grad}}(T) \tag{1}$$

Where:

- $\vec{\phi}$:The heat-flow density (w.m^{-2}).
- T : Temperature(K).
- λ : Thermal conductivity (W.m^{-1}.k^{-1}).

Let us consider a cylindrical medium, homogeneous, of section S and length L, the two sides of the cylinder are maintained respectively at temperatures T_1 (cold source) and T_2 (hot source) [Fig1]. There is a transfer of energy from the hot source to the cold source. Since the medium is homogeneous, the temperature is evenly distributed in a steady state [15].

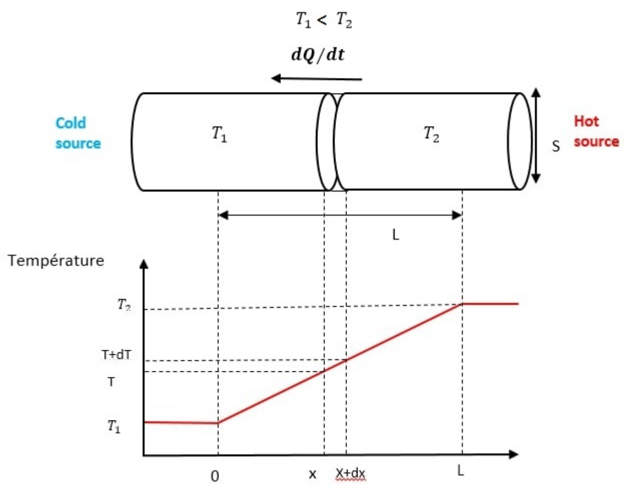

Figure 1 – *Cylindrical medium maintained at different temperatures*

On a permanent basis, the Fourier law expresses the amount of elemental heat dQ that passes through x a surface S of thickness dx during time dt.
Fourier's law can be written in the form:

$$\frac{\delta Q}{\delta t} = -\lambda S \frac{dT}{dx} \tag{2}$$

Where:

- δQ : Elemental energy(J).
- δt : Elementary time(s).
- λ : Thermal conductivity (W.m^{-1}.k^{-1}).
- S : Section (m^2).
- T : Temperature(k).
- x : Spatial co-ordinate.

III. Economic Parenthesis: Supply and Demand

On a market, supply and demand exist at the same time; it's their interaction that will make a price stabilize on the market and ensure that a certain amount of the good will be buying and selling out [16, 17].

In the figure below [Fig 2] (the x-axis of this graph represents quantity Q and the y-axis stands for price P, the price on the market is P_e. At this price, the quantities offered Q_s are equal to the quantities demanded Q_d and both of them are equal to the equilibrium about Q_e : We say that the market is in equilibrium. Any other point than the equilibrium point E would be a point of imbalance in the market. The forces of supply and demand will ensure that there is always an attempt to return to point E [16, 17].

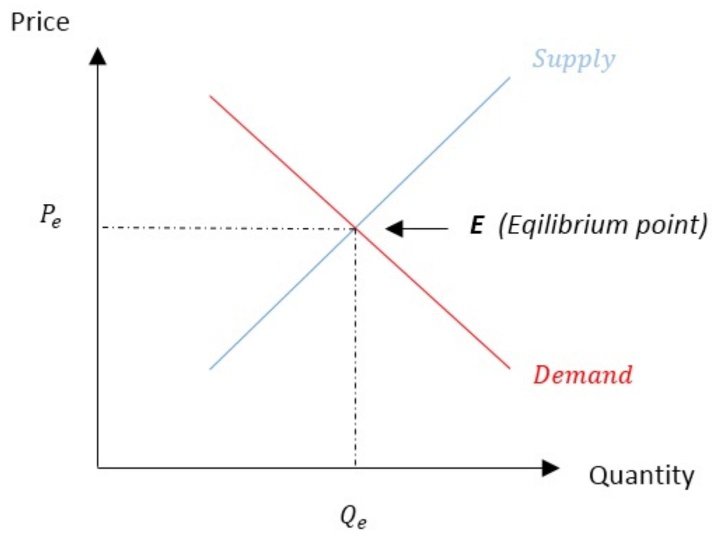

Figure 2 – *Equilibrium point between quantity offered and requested in term of price*

This curve illustrates:

- The Demand which is the quantity of products that consumers are ready to buy it for a certain price. They were willing to buy more (i.e. a higher quantity) of a good or service if the price falls. So for every price there is a quantity demanded, which will be higher the lower the price is [16, 17].
- Supply, which is the quantity of products that producers seek to sell to a given price. Since the price associated with a quantity offered depends on the cost of production and that producers seek to maximize the gains, the quantity offered is a growing function of the price. This confrontation of supply and demand leads to an equilibrium price, which is the price that makes demand and supply equal [16, 17].
- The point where both curves (demand and supply) intersect is called the market equilibrium E. At this point the consumers are willing to buy exactly as much of a good or service as the producers are willing to sell. This is the best possible situation for all actors, thus they will always tend to get to this outcome. This means the two curves will keep shifting until the equilibrium quantity and price are reached [16, 17].

IV. Adaptation to the economic equilibrium

This section is about issuing a thinking on the confrontation of economic and physical theories. The cursor will be placed essentially on the principle of economic equilibrium by analogy with thermodynamics[18] . In [3] François Roddier describes economic exchanges by assigning to each production two potentials similar to Gibbs Duhem's potentials. Either the function of Gibbs given by:

$$G = -PdV + TdS \tag{3}$$

The pressure and temperature in partial derivative is given by the equations:

$$P = \frac{\partial G}{\partial V}, \tag{4}$$

$$T = \frac{\partial G}{\partial S}. \tag{5}$$

where

- G : Free entalpy(J).
- S : Entropy(J/K).
- V : Volume(m^3).
- T : Temperature(K).
- P : Pressure(Pa).

In[3] we find the following economic interpretation:

- P : Utility assigned to certain production.
- T : Price of production.
- V : Production volume.

- S : Monetary contribution associated with dV.

Based on the attribution of the thermal temperature to the notion of price, we present in this contribution, a reflection about the use of thermodynamics in the understanding of the processes of supply and demand. In thermodynamics, the experience proves that during a heat exchange between two solids 1 and 2 with different temperatures, their temperatures converges towards an equilibrium temperature, then the equilibrium situation is characterized by the following equation:

$$Q_1 = Q_2 \tag{6}$$

Where :

- Q_1 is the quantity of heat in the solid 1
- Q_2 is the quantity of heat in the solid 2

Indeed consider a solid of uniform temperature T_1, at a time t we put it in contact with another solid with temperature T_2 such as $T_1 > T_2$. As soon as the two bodies are put into contact, the system is no longer at equilibrium. It evolves towards a state of equilibrium characterized by a temperature T_e. We say that there is an energy transfer by "heat." The temperature of the hot body decreases and the temperature of the cold body increases (Famous principle of thermodynamics) (4). The following figure shows the profile of the temperature in both body at the beginning, during the transformation, and at the final equilibrium[Fig 3].

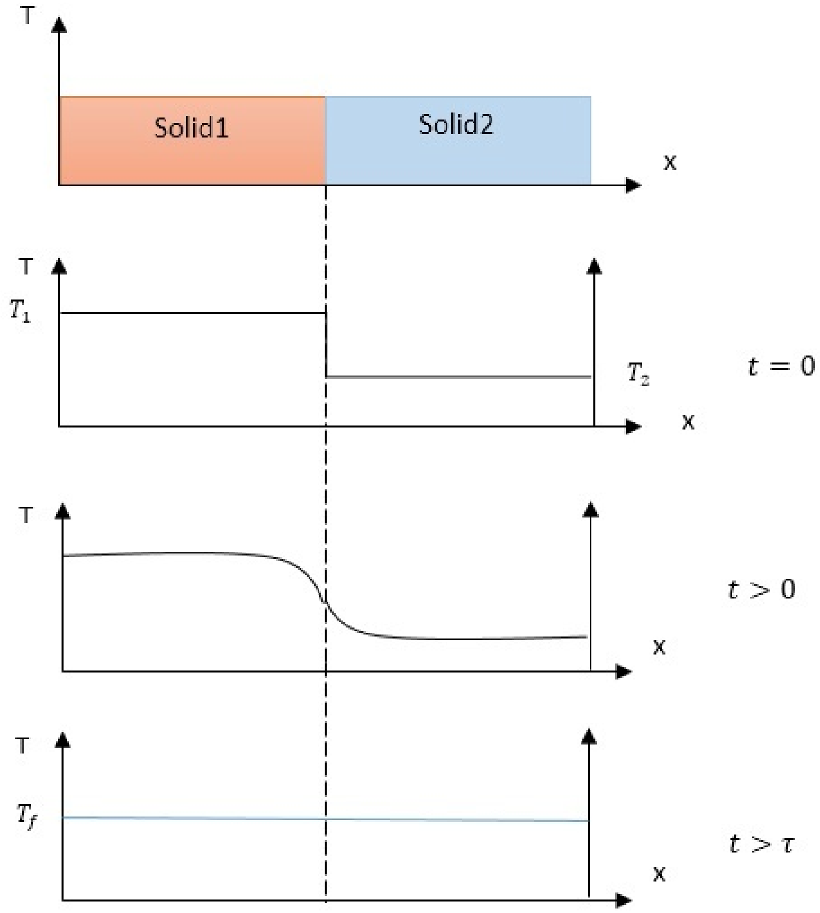

Figure 3 – *The temperature profile of hot and cold bodies*

Based on these standard notions of thermodynamics, we believe that in economic language, it is possible to propose the following interpretation. on a market of pure and perfect competition in a situation of imbalances, in order to reach the price equilibrium, the quantity required must be reduced (hot body loses heat) as to the quantity offered must be increased (cold body receives heat) in order to reach a price level that balances the quantity offered and the quantity demanded [Fig 2].
Then the market situation can be described by the equation:

$$Q_s = Q_d \tag{7}$$

Where :

- Q_s is the quantity of supply
- Q_d is the quantity of demand.

i. Positive offer shock

In thermodynamics, a hot body and a cold one in a state of equilibrium, if we add a third one, the system converges to a new state of equilibrium.
Applied to economic thought, in a balanced market where supply and demand are in balance, the unit price in a batch of products (called hot) exposed for sale is set according to the law of supply and demand. On arrival of a new batch (called cold) of the same product, a collision with the old batch will be generated and will destabilize the market so that supply be greater than the demand. As a result, the price of products in the first batch will fall but the second batch will win in terms of price in order to recreate lost balance. At this stage, the price is equal to the equilibrium price defined by "supply = demand". In macroeconomic language this phenomenon is called the positive supply shock.
In fact, let's start from an initial equilibrium situation (p_1, q_1)[Fig 4] and make the hypothesis for example a modification of the supply. Under the effect of a technological improvement which allowed companies to produce more at lower prices, supply tends to increase (we go from the supply curve S_2 to the supply curve S_1) and the wanted price decreases by consequence in order to establish a new equilibrium price which equals the supply and demand.

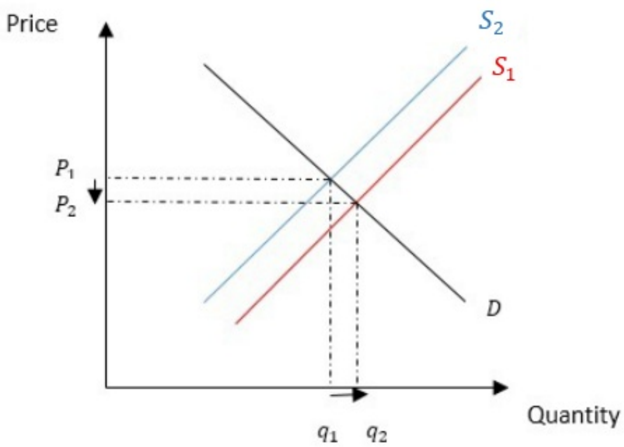

Figure 4 – *The positive supply shock*

ii. The microscopic scale

At the microscopic scale, conductive heat transfer is the origin of a specific phenomenon. So, it is the kinetic energy of the molecules which generates the thermal transfer by conduction because of the shocks made in the area of contact, thus causing the increase of the microscopic kinetic energy of the cold body particles, this is called a transfer of energy by "heat" [14].

The phenomenon described above is the equivalent of the collision phenomenon realized between the consumer and the producer, while they are regarded as agitated molecules during a sale or purchase transaction. If during the transfer of energy by heat, the molecules of the hot body lose energy and their cold-body equivalents gain it, that's exactly the same operation during a sale and purchase transaction, one agent loses his money while the other recover it.

iii. Fourrier's law in economics

iii.1 Conductivity

The Experience in thermodynamics shows that solids do not conduct thermal energy in the same way. Indeed, by heating four different metal blades simultaneously, we find that the ability to conduct heat differs from one metal to another1.

Table 1 – *Thermal conductivity for certain material [19]*

Material	**Conductivity**(W$m^{-1}k^{-1}$)
Tungsten	196,65
Aluminum	225,94
Gold	317,98
Copper	397,48
Silver	429,77

In an economic context, where supply and demand intersect to reach an equilibrium price, we can define an economic conductivity λ_e. This length measures the purchasing power of consumers in this market, in other words, it characterizes a market's ability to balance while remaining in the proximity of P_i (Initial price). As a matter of fact, let's consider a situation where the demand for a product is more important than its offer, a new product that is launched simultaneously in two markets such as the purchasing power in the market 1 is greater than the purchasing power in the second (zone 1) [Fig 5]. The product is launched at a single price P_i, if we set the amount of balance q_e, we will notice that the market 1 reaches the equilibrium point (q_e, p_1) before the market 2 which equilibrium point is (q_e, p_2), in other words, the consumer in the market 1 is willing to acquire a product in exchange for a price that the consumer on the second market is not able to pay (depending on its income for example), we say that market 1 is more conducive than market 2. In another way we can put forward the idea that the quantum number associated with the conduction of markets is the purchasing power (PP).

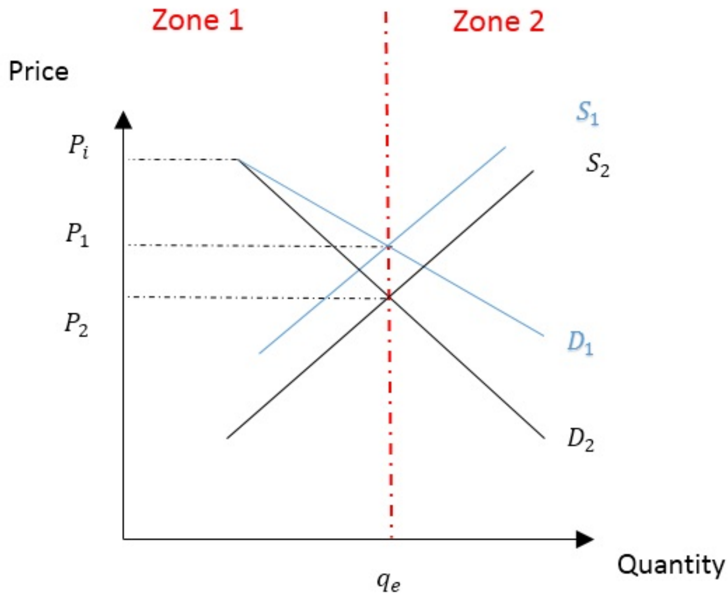

Figure 5 – *Two situations of economic equilibrium*

iii.2 Economic flow

We will consider an economic market for which the demand exceeds offers.
Let's get into the situation where we have a quantity offered at the price P_1 and quantity asked for the price P_2; in order to reach the equilibrium price, the quantity offered must be increased as for the quantity requested it must be reduced. As a consequence of this displacement of quantities in time, a flow, called product flow is created in the same way as the heat flow in thermodynamics $\Phi = \frac{dQ}{dt}$.

This phenomenon of variation of quantity offered and requested also creates a price gradient $\frac{dp}{dx}$, this gradient is quantified by the derivative of price with respect to the number of economic agents participating in the market outcome. The graph above[Fig 6] shows that the variation of the quantity of products over time causes a price change depending on the market size (MS); defines as a number of economic agents in the market. It can therefore be concluded that there is a relationship of proportionality between the product flow and the price gradient, so we can postulate the following equation.

$$\frac{d\delta Q_p}{\delta t} = -a\frac{dP}{dx} \tag{8}$$

Where:

- δQ : The elementary quantity of the product.
- δt : The elementary time(s).
- a : The gradient of the supply curve (in physical language a=-λ S).
- P : The price.

- x : The market size.

We notice that this is exactly the Fourier equation in the equation (5)

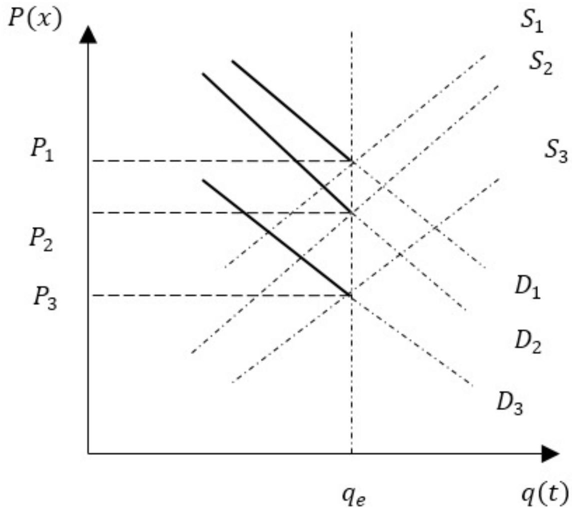

Figure 6 – *Price gradient variation in function of product flow*

It is proposed to establish the following analogies:

Table 2 – *Corresponding terms in economics and thermodynamics.*

Symbol	**Economics**	**Unit**	**Symbol**	**Thermodynamics**	**Unit**
P	Price	\$,€	T	Temperature	k
Q	Product Quantity	-	Q	Heat Quantity	J.$Kg^{-1}.k^{-1}$
PP	Purchasing Power	-	λ	Thermal conductivity	W.$m^{-1}.k^{-1}$
MS	Market Size	-	X	Spatial Coordinate	-
t	Time	s	t	Time	s

V. Conclusion

To conclude, we recall that throughout this work, which is part of a Following logical research work with multidisciplinary profile, we tried to bolster up the recent axis of econophysics.

It would be exciting in future work to study, following the same approach, the variation of price in markets of imperfect competition such as the market in a monopoly and oligopoly situation. Another concept also to analyze, using the stationary regime in order to determine the economic resistance. On the other side we can use the first principle of thermodynamics to establish a partial derivatives equation of the economic diffusion of price with respect to time and to the size of the market.

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2} \tag{9}$$

Where a is the the economic diffusivity coefficient.

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